Simple conditions for L_2 stability and stabilization of networked control systems

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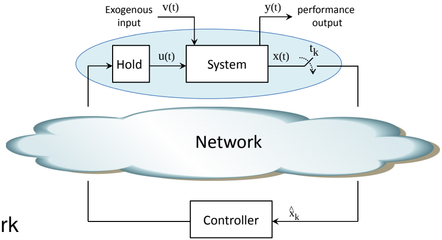
Outline

- Introduction
- Problem statement
- Stability analysis
- Control
- Conclusion and Future Works

NCS



- Remote control
- Wireless network
  - Data loss
  - Time-varying propagation delays
  - Varying sampling period

Existing approaches

- Time-delay systems [Yu et al.], [Fridman et al.]
- Impulsive systems [Naghshtabrizi et al.], [Seuret]
- Sampled-data systems [Mirkin]
- Robust techniques [Fujioka], [Oishi et al.]
- Functional-based approaches [Seuret]

Problem statement

NCS model

- Process model

$$\begin{array}{rcl} \dot{x}(t) & = & Ax(t)+Bu(t)+Ev(t) \\ y(t) & = & Cx(t)+Du(t)+Fv(t) \\ x(0) & = & x_0 \end{array} \tag{1}$$

- Control-law model

$$\begin{array}{rcl} u(t) & = & Kx(t_k) \\ t & \in & [t_k, t_{k+1}) \\ t_{k+1}-t_k & \leq & (1+m)T_{max}+\tau_{k+1} \\ \tau_k & \in & [0, \tau_{max}] \end{array} \tag{2}$$

  - t_k: arrival instants of a new control input
  - $\tau_k = \tau(t_k)$, $k \in \mathbb{N}$
  - m is the number of consecutive dropouts
- Varying sampling period $\rightarrow$ Actual (varying) sampling period+data loss+varying propagation delays

Problem

- Find a sampled-data state-feedback control law such that the closed-loop system is asymptotically (exponentially) stable and

  objective 1: maximize the Maximal Allowable Transfer Interval (MATI) under L_2 disturbance attenuation constraints; or
  objective 2: minimize L_2 disturbance attenuation gain under a MATI constraint.

$$\mathscr{S}_{\text{MATI}} := \{(m, \tau, T) \in \mathbb{N} \times \mathbb{R}_+ \times \mathbb{R}_{++} : \ (1+m)T + \tau \leq \text{MATI}\}. \tag{3}$$

- m: number of consecutive dropouts
- T: sampling period
- τ: propagation delay

Stability Analysis

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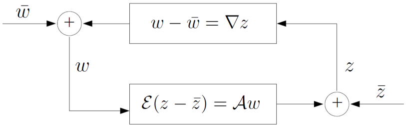
Quadratic separation



Theorem

The interconnected system above is well-posed if there exists a symmetric matrix Θ satisfying the conditions

$$\begin{bmatrix} \mathcal{E} & -\mathcal{A} \end{bmatrix}_\perp^T \Theta \begin{bmatrix} \mathcal{E} & -\mathcal{A} \end{bmatrix}_\perp \succ 0 \tag{4}$$

and

$$\left\langle \begin{bmatrix} 1 \\ \mathbb{P}_T \nabla \end{bmatrix} u_T, \Theta \begin{bmatrix} 1 \\ \mathbb{P}_T \nabla \end{bmatrix} u_T \right\rangle \leq 0 \tag{5}$$

for all $u \in L_{2e}$ and all $T > 0$.

Alternative system representation

$$\underbrace{\begin{bmatrix} x(t) \\ \delta(t) \\ v(t) \end{bmatrix}}_{w(t)} = \underbrace{\begin{bmatrix} \mathcal{I}\mathrm{1_n} & & \\ & \Delta_{sh}\mathrm{1_n} & \\ & & \Delta_\gamma \end{bmatrix}}_{\nabla} \underbrace{\begin{bmatrix} \dot{x}(t) \\ \dot{x}(t) \\ y(t) \end{bmatrix}}_{z(t)}, \tag{6}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathcal{E}} \underbrace{\begin{bmatrix} \dot{x}(t) \\ \dot{x}(t) \\ y(t) \end{bmatrix}}_{z(t)} = \underbrace{\begin{bmatrix} A+BK & -BK & E \\ 0 & 0 & 0 \\ C+DK & -DK & F \end{bmatrix}}_{\mathcal{A}} \underbrace{\begin{bmatrix} x(t) \\ \delta(t) \\ v(t) \end{bmatrix}}_{w(t)} \tag{7}$$

- $\mathcal{I}$: integral operator,
- $\Delta_{sh} : \theta \to \int_{t_k}^{t} \theta(s)ds,\ t \leq t_{k+1} \Rightarrow \delta(t) = x(t) - x(t_k)$.
- Δ_γ: virtual operator characterizing the L_2 gain of the transfer $v \to y$.

IQC for the integral operator

Lemma

The integration operator $\mathcal{I}$ is characterized by the IQC:

$$\left\langle \begin{bmatrix} \mathrm{1_n} \\ \mathcal{I}\mathrm{1_n} \end{bmatrix} x_T, \begin{bmatrix} 0 & -P \\ -P & 0 \end{bmatrix} \begin{bmatrix} \mathrm{1_n} \\ \mathcal{I}\mathrm{1_n} \end{bmatrix} x_T \right\rangle \leq 0.$$

for all $x \in L_{2e}^n$ and for any matrix $P \in \mathbb{S}_{++}^n$.

- Lyapunov condition for stability
- Frequency domain $\mathcal{I} \to s^{-1}$

$$\begin{bmatrix} \mathrm{1_n} \\ s^{-1}\mathrm{1_n} \end{bmatrix}^* \begin{bmatrix} 0 & -P \\ -P & 0 \end{bmatrix} \begin{bmatrix} \mathrm{1_n} \\ s^{-1}\mathrm{1_n} \end{bmatrix} \preceq 0, \ \forall \Re[s] \geq 0$$

Pre- and post-multiply by $s^* X(s)^*$ and $sX(s)$

$$-(s+s^*)X(s)^* P X(s) \preceq 0, \ \forall \Re[s] \geq 0 \quad (8)$$

Characterization of all $\Re[s] \geq 0$: pick any $P = P^T \succ 0$

IQC for Δ_{sh}

Lemma

The operator Δ_{sh} can be characterized by the IQC:

$$\left\langle \begin{bmatrix} 1_n \\ \Delta_{sh} 1_n \end{bmatrix} x_T, \begin{bmatrix} -\frac{4}{\pi^2}\mu^2 S_1 & -S_2 \\ -S_2 & S_1 \end{bmatrix} \begin{bmatrix} 1_n \\ \Delta_{sh} 1_n \end{bmatrix} x_T \right\rangle \leq 0.$$

for all $x \in L_{2e}^n$ and for any matrices $S_1, S_2 \in \mathbb{S}_{++}^n$.

- S_1: Bound on the L_2-gain of $2\mu/\pi$, $t_{k+1} - t_k \leq \mu$, $k \in \mathbb{N}$ [Mirkin]
- S_2: Passivity of Δ_{sh} [Fujioka]

IQC for Δ_γ

Lemma

The operator Δ_γ, which has an L_2-induced norm equal to γ^{-1}, is characterized by the IQC:

$$\left\langle \begin{bmatrix} 1_r \\ \Delta_\gamma \end{bmatrix} x_T, \begin{bmatrix} -\gamma^{-2}1_q & 0 \\ 0 & 1_r \end{bmatrix} \begin{bmatrix} 1_q \\ \Delta_\gamma \end{bmatrix} x_T \right\rangle \leq 0,$$

for all $x \in L_{2e}^q$.

- Upper bound γ on L_2-gain of $v \to y$

Stability result

Theorem

The NCS system is asymptotically stable for all $(m, \tau, T) \in \mathscr{S}_\mu$ *if there exist matrices* $P, S_1, S_2 \in \mathbb{S}^n_{++}$ *and a scalar* $\eta > 0$ *such that the LMI*

$$\begin{bmatrix} \mathcal{E} & -\mathcal{A} \end{bmatrix}_\perp^T \Theta \begin{bmatrix} \mathcal{E} & -\mathcal{A} \end{bmatrix}_\perp \prec 0 \tag{9}$$

holds where $\mathcal{E}, \mathcal{A}$ *are defined in (7) and*

$$\Theta = \left[\begin{array}{ccc|ccc} 0 & 0 & 0 & -P & 0 & 0 \\ 0 & -\frac{4}{\pi^2}\mu^2 S_1 & 0 & 0 & -S_2 & 0 \\ 0 & 0 & -\eta 1_q & 0 & 0 & 0 \\ \hline & & & 0 & 0 & 0 \\ & * & & 0 & S_1 & 0 \\ & & & 0 & 0 & 1_r \end{array}\right]. \tag{10}$$

Moreover, the closed-loop system satisfies $||y||_{L_2} \leq \sqrt{1/\eta}||v||_{L_2}$.

Example

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} -1.006 & -1.006 \end{bmatrix} x(t_k). \quad (11)$$

- Maximal constant sampling period: 5.8117.

	$MATI$	nb. of vars.	for $n = 2$
[Yu, 04]	unfeasible	$4\frac{n(n+1)}{2}$	12
[Yue, 04]	0.970	$2\frac{n(n+1)}{2} + 6n^2$	30
[Tan, 08]	0.995	$4\frac{n(n+1)}{2} + 16n^2$	76
[Naghshtabrizi, 06](without delay)	1.272	$7\frac{n(n+1)}{2} + 16n^2$	85
Proposed result	1.561	$3\frac{n(n+1)}{2} + 1$	10

Stabilization

Main result

Theorem

There exists a matrix $K \in \mathbb{R}^{m\times n}$ such that the NCS is asymptotically stable for all $(m,\tau,T) \in \mathscr{S}_\mu$ if there exist matrices $P, S_1 \in \mathbb{S}^n_{++}$, $X \in \mathbb{R}^{n\times n}$, $U \in \mathbb{R}^{m\times n}$ and a scalar $\gamma > 0$ such that the LMI

$$\begin{bmatrix} -(X+X^T) & P+A'_{cl} & -BU & E & 0 & X & \mu\frac{\pi}{2}S_1 \\ \star & -P & 0 & 0 & {C'_{cl}}^T & 0 & 0 \\ \star & \star & -S_1 & 0 & -(DU)^T & 0 & 0 \\ \star & \star & \star & -\gamma I & F^T & 0 & 0 \\ \star & \star & \star & \star & -\gamma I & 0 & 0 \\ \star & \star & \star & \star & \star & -P & -\mu\frac{\pi}{2}S_1 \\ \star & \star & \star & \star & \star & \star & -S_1 \end{bmatrix} \prec 0 \quad (12)$$

holds with $A'_{cl} = AX + BU$ and $C'_{cl} = CX + DU$. Furthermore, the closed-loop system controlled with gain $K = UX^{-1}$ satisfies $||y||_{L_2} \leq \gamma||v||_{L_2}$.

Example

- Let us consider the open-loop system

$$\dot{x}(t) = \begin{bmatrix} -0.8 & -0.01 \\ 1 & 0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0.4 \\ 0.1 \end{bmatrix} u(t) \tag{13}$$

- [Yu, 04]: system stabilizable for $\mu \leq 0.6011$.
- Proposed result: system stabilizable for $\mu \leq 3.64826$ with the controller gain $K = \begin{bmatrix} -0.3482 & -0.3097 \end{bmatrix}$.

Conclusion

- Approach based on well-posedness and IQCs
- LMI results for both stability and stabilization
- Tradeoff: L_2-gain minimization vs. MATI maximization
- Low numerical complexity
- Can be extended to robust stability and stabilization

Thank you for your attention