The conservation of information and the congestion control modeling problem

Corentin Briat

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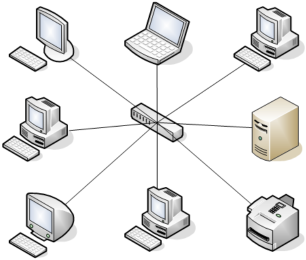
Introduction

Fundamentals on communication

Network Elements

- Buffers/Servers/Routers (queuing delays, loss)
- Transmission channels (delays, limited capacity)
- Users (senders, receivers)



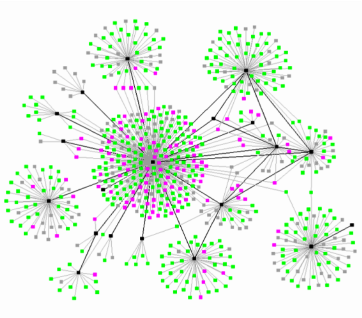
Transmission principle

- Information split into packets
- User A sends packets destined to User B
- Packets routed through the network
- User B receives packets and acknowledges them (ACK packet)
- User A receives ACK packets $\rightarrow$ transmission successful

Motivations

Congestion problem

- The users (green) communicate all together
- Packets accumulate at the servers (black)
- Large delays, data loss
- Congestion control



Why congestion modeling ?

- Understanding the process of congestion
- Simulation purpose
- Protocol validation/design

Congestion models

Time-domain

- Discrete-time models [Johari et al., 01], [Shorten et al., 06]
- Continuous-time models [Paganini et al., 05], [Vinnicombe, 00] $\rightarrow$ fluid-flow model.
- ★ Hybrid models [Hespanha et al., 01]

Stochastic vs. Deterministic

- Stochastic [Misra et al., 00]
- ★ Deterministic [Jacobsson et al., 08], [Tang et al., 10]

Modular vs. monolithic

- Monolithic [Misra et al., 00], [Hollot et al., 01]
- ★ Modular [Paganini et al., 05], [Liu et al., 07]

What properties a network model should have ?

1. Few universal concepts (quantities) related by laws.
2. Local description of the elements (modularity, scalability)
3. New models corresponding to new devices may be freely added without compromising existing ones (genericity).
4. Easy transcription of the network into a topologically identical diagram/model, and vice-versa.
5. Model predictions fit the reality.
6. Systematic way of analysis by hand calculations or simulators.

Modeling procedure

Core principle

- The conservation law of information

Block model derivation

- Transmission channels
- Buffers/Queues
- Users

Desired properties for the model

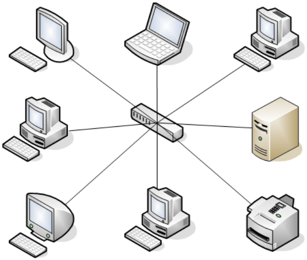
- Explicit, modular, scalable and accurate
- Similarities with electrical networks models

Network congestion modeling

Fluid-flow models

Ingredients

- Flight-size: controlled variable
- Congestion window: reference
- Congestion measure: measurement
- Sending rate: control input



Rate/flow definition

- Quantity of information: N [bit] or [Pkt]
- Rate: ϕ [bit/s] or [Pkt/s]
- Quantity of information having passed through point x between t_0 and t:

$$N_x(t_0, t) := \int_{t_0}^{t} \phi(x, s)ds$$

The conservation law of Information

Observations

- We can count the number of packets $P_E(t)$ in an element E (e.g. a queue) at time t simply by counting the entering packets over a certain horizon

The conservation law of information

There exists $t_0(t) \leq t$ such that the following equality holds:

$$P_E(t) \quad = \quad \int_{t_0(t)}^{t} \phi(s)ds.$$

where $\phi(t)$ is the input flow of the element E.

- Particular case of this equation (ACK-clocking model) proposed in [Jacobsson,08]

If you have to remember something from this talk, just remember the above equation !

Balance equation and output flow computation

Balance equation

- Variation of the number of packets in element E

$$
\begin{array}{rcccc}
P_E'(t) & = & \overbrace{\phi(t)}^{\text{input flow}} & - & \overbrace{\phi^o(t)}^{\text{output flow}} \\
 & = & \phi(t) & - & t_0(t)'\phi(t_0(t))
\end{array}
$$

Output flow model

The output flow of element E, $\phi^o(t)$, is given by

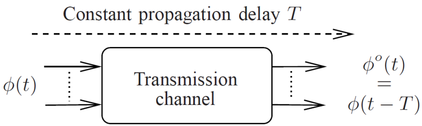
$$\phi^o(t) := t_0(t)'\phi(t_0(t)).$$

- Delay
- Amplitude distortion

Transmission Channel model

Assumptions

- Lossless
- Constant propagation delay $T > 0$



Conservation law for transmission channels

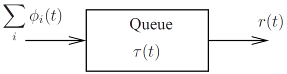
$$\begin{aligned} t_0(t) &= t - T \\ P_E(t) &= \int_{t-T}^{t} \phi(s)ds \end{aligned}$$

Balance equation and output flow expression

$$\begin{aligned} P'_E(t) &= \phi(t) - \phi(t-T) \\ \phi^o(t) &= \phi(t-T) \end{aligned}$$

Basic queue model

$$\begin{aligned}\dot{\tau}(t) &= \frac{1}{c}\left(\sum_j \phi_j(t) - r(t)\right)\\ r(t) &= \begin{cases} c & \text{if } \tau(t) > 0 \text{ or } \sum_j \phi_j(t) \geq c \\ \sum_j \phi_j(t) & \text{otherwise} \end{cases}\end{aligned}$$



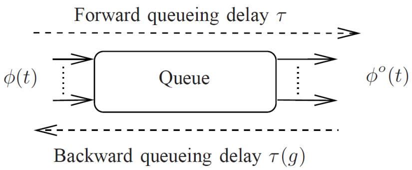
Buffer model

- Queuing delay $\tau(t)$
- Maximal output capacity c

Model disadvantages

- Aggregate output flow $r(t)$! How to split it ?
- Does this model really capture the behavior of a FIFO queue ?

Queue model

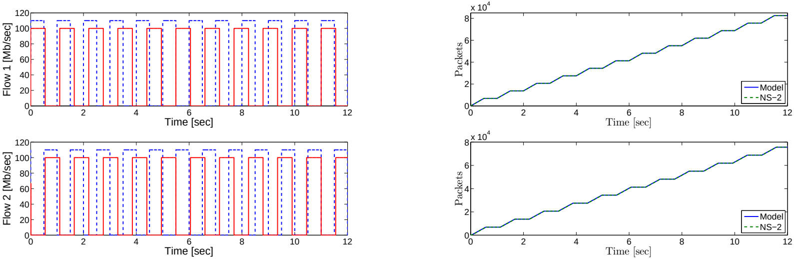


FIFO buffer model

$$\begin{aligned}
\dot{\tau}(t) &= \frac{1}{c}\sum_j \left[\phi_j(t) - \phi_j^o(t)\right] \\
\phi_j^o(t) &= \begin{cases} \dfrac{\phi_j(t_0(t))c}{\sum_k \phi_k(t_0(t))} & \text{if } \tau(t) > 0 \text{ or } \displaystyle\sum_j \phi_j(t) \geq c \\ \phi_j(t) & \text{otherwise} \end{cases} \\
t_0(t) &= t - \tau(t_0(t))
\end{aligned}$$

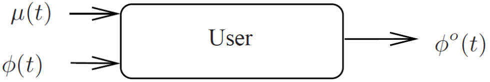
- Model proposed in [Ohta et al., 98], [Liu et al., 04], without proof
- We see here that it is an immediate consequence of the conservation law

Example - One queue/two flows



- Single queue, two on/off input flows in phase opposition
- Exact matching with NS2

Complete user model



Protocol Equations

$$
\begin{array}{lrcl}
\text{Congestion measure} & \mu(t) & = & f(\tau(t)) \\
\text{Protocol state} & \dot{z}(t) & = & \mathcal{P}(z(t), \mu(t)) \\
\text{Congestion window} & w(t) & = & \mathcal{W}(z(t), \mu(t))
\end{array}
$$

Sending rate model

sending rate

$$
\phi^o(t) = \begin{cases} \dot{w}_i(t) + \phi(t) & \text{if } \mathcal{T}_i(t) \\ 0 & \text{otherwise} \end{cases}
$$

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flight-size control

$$
\dot{\pi}_i(t) = \begin{cases} 0 & \text{if } \mathcal{T}_i(t) \\ \dot{w}_i(t) + \phi(t) & \text{otherwise} \end{cases}
$$

$$
\mathcal{T}_i(t) = ([\pi_i(t) = 0] \wedge [\dot{w}_i(t) + \phi(t) \geq 0])
$$

Model benefits

Structure

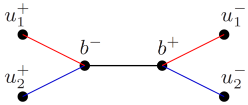
- Coherence of the blocks: one interconnection variable, local description variables (delays, congestion windows, etc.)
- Explicit, modular and scalable representation

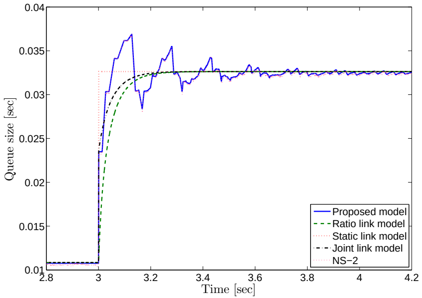
Theoretical implications

- Previous rate models are approximations of the conservation law (ACK-clocking model)
  - Ratio model $\phi^o(t) \simeq w(t)/\,\mathrm{RTT}\{t\}$ [Paganini et al., 02], [Vinnicombe, 00]
  - Joint model $\phi^o(t) \simeq w(t)/\,\mathrm{RTT}\{t\} + \dot{w}(t)$ [Jacobsson et al., 08]
- Static model $\phi^o(t) \simeq \dot{w}(t)$ [Wang et al., 05] can be shown to be exact for some particular network topologies.
- Window-based ACK-clocking model $w(t) \simeq P_C(t)$ [Tang et al., 10] exact when the packet counter is always at 0

Examples

Single buffer - Two Users

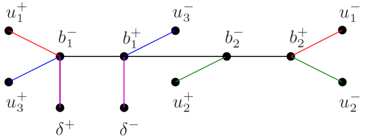


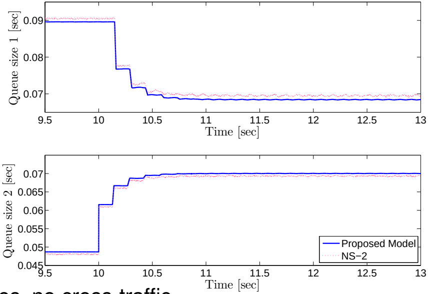


Scenario 1 [Tang et al., 10]

- $c = 100$Mb/s, $\rho = 1590$ bytes
- Propagation delays: $T_1 = 3.2$ms and $T_2 = 117$ms
- Initial congestion window sizes: $w_1^0 = 50$ and $w_2^0 = 550$ packets
- At 3s, w_1 is increased to 150 packets.

Two buffers - Three Users





Scenario 2 [Tang et al., 10]

- $c_1 = 72$Mb/s, $c_2 = 180$Mb/s, $\rho = 1448$ bytes, no cross-traffic
- Propagation delays: $T_1 = 120$ms, $T_2 = 80$ms and $T_3 = 40$ms
- Initial congestion window sizes: $w_1^0 = 1600$, $w_2^0 = 1200$ and $w_3^0 = 5$ packets
- At 10s, w_2 is increased to 1400 packets.

Conclusion

Conclusion

- Model built upon one fundamental principle: the conservation of information
- Model is modular, scalable, topologically identical
- Provides insights on validity domains of flow models
- Describe quite well the reality for considered topologies
- Suitable for building (graphical) simulators

Current Works

Linear model

- Equilibrium point (w^*, ϕ^*, τ^*) computed by solving a convex optimization problem

Transmission channel model

$$\phi^o(t) = \phi(t - T)$$

Queue model

$$\begin{aligned} \dot{\tau}(t) &= \frac{1}{c}\sum_i \phi_i(t) \\ \phi^o(t) &= \underbrace{\left(I - \frac{1}{c}\phi^* \mathbb{1}^\intercal\right)}_{\text{Competitive matrix}} \phi(t - \tau^*) \end{aligned}$$

User model

$$\begin{aligned} \dot{z}(t) &= (Az)(t) + B\mu(t) \\ w(t) &= (Cz)(t) \\ \phi^o(t) &= \dot{w}(t) \end{aligned}$$

A simple local stability result - single user/single queue

FAST-TCP protocol (CT approximation)

$$\begin{array}{lcl} \dot{w}(t) & = & \dfrac{\log(1-\gamma)}{T+\mu(t)}\left(-w(t)+\dfrac{T}{T+\mu(t)}w(t-\mu(t)-T)+\alpha\right) \\ \mu(t) & = & \tau(g(t-T_b)) \\ T & = & T_b+T_f \end{array} \quad (1)$$

Stability result

The network is locally exponentially stable for all $c > 0, T > 0, \gamma > 0, \alpha \geq 1$ and any constant cross-traffic $\delta^* \in [0, c)$.

Extensions

- Single buffer/multiple users: in progress
- Multiple buffers/multiple users: nice results possible for some given topologies, e.g. triangular one
- Input/output approaches may be suitable: modular analysis tools for modular models...

Thank you for your attention