Stability Criteria for Asynchronous Sampled-data Systems - A Fragmentation Approach

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Outline

- Introduction
- Problem statement and Preliminaries
- Stability analysis
- Conclusion and Future Works

Aperiodic sampled-data systems

- Discrete-time systems with varying sampling period
- Several frameworks
  - Time-delay systems [Yu et al.], [Fridman et al.]
  - Impulsive systems [Naghshtabrizi et al.], [Seuret]
  - Sampled-data systems [Mirkin]
  - Robust techniques [Fujioka], [Oishi et al.], [Ariba et al.]
  - Functional-based approaches [Seuret]

Problem statement

System and Problem definition

- Continuous-time LTI system

$$\begin{array}{rcl} \dot{x}(t) & = & Ax(t) + Bu(t) \\ x(0) & = & x_0 \end{array} \quad (1)$$

with state x and control input u.

- Sampled-data control law

$$u(t) = Kx(t_k), \ t \in [t_k, t_{k+1}) \quad (2)$$

where $T_k := t_{k+1} - t_k \in \mathcal{T} := [T_{min}, T_{max}]$, $k \in \mathbb{N}$.

- Stability analysis problem: given K, find the set $\mathcal{T}$ for which for all $T_k \in \mathcal{T}$ stability holds.

Quadratic stability result

- It is straightforward to show that the system is asymptotically stable if there exists $P = P^T \succ 0$ such that the LMI

$$\Phi(T)^T P \Phi(T) - P \prec 0$$

for all $T \in \mathcal{T}$ and where

$$\Phi(T) = e^{AT} + \int_0^T e^{A(T-s)} ds BK \tag{3}$$

- LMI difficult to check (although possible [Fujioka])
- Difficult to extend to uncertain systems or nonlinear systems
- Alternative way ?

Alternative discrete-time stability condition

Theorem

Let $V(x) = x^T P x$, $P = P^T \succ 0$, P finite and define $\varkappa_k(\tau) := x(t_k + \tau)$, $\chi_k \in C([0, T], \mathbb{R}^n)$, $k \in \mathbb{N}$. Then the two following statements are equivalent:

(i) *The LMI $\Phi(T)^T P \Phi(T) - P \prec 0$ holds for all $T \in \mathcal{T}$.*

(ii) *There exists a continuous functional $V_1 : \mathbb{R} \times C([0, T], \mathbb{R}) \to \mathbb{R}$, differentiable over $[t_k\ t_{k+1})$ satisfying*

$$V_1(T_k, \varkappa_k) = V_1(0, \varkappa_k) \quad (4)$$

for all $k \in \mathbb{N}$ and such that the functional

$$\mathcal{W}(\tau(t), \varkappa_k) := V(x(t)) + V_1(\tau(t), \varkappa_k(\tau(t)))$$

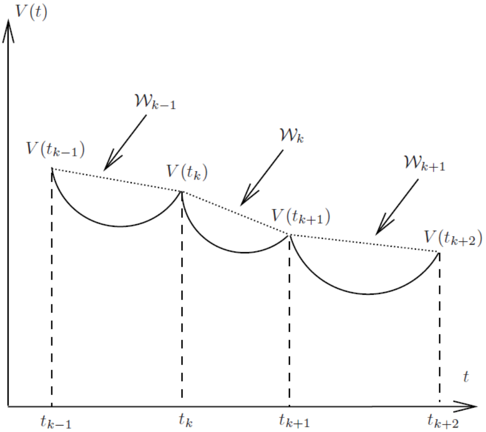
satisfies

$$\dot{\mathcal{W}}(\tau(t), \varkappa_k) = \frac{d}{dt}\mathcal{W}(\tau(t), \varkappa_k) < 0 \quad (5)$$

for all $\tau \in [0, T_k]$, $T_k \in \mathcal{T}$, $k \in \mathbb{N} - \{0\}$.

Moreover, if one of these two statements is satisfied, the solutions of the sampled-data are asymptotically stable.

Illustration of the result



Connection with impulsive approach

- In the impulsive framework [Naghshtabrizi et al.], [Seuret], the functional may be considered

$$
\begin{aligned}
V &= x(t)^T P x(t) + (T_k - \tau)(x(t) - x(t_k))^T S (x(t) - x(t_k)) \\
&\quad + (T_k - \tau) \int_{t_k}^{t} \dot{x}(s)^T R \dot{x}(s) ds, \ t \in [t_k, t_{k+1}]
\end{aligned}
$$

  where $\tau = t - t_k$ and P, S, R symmetric positive definite.
- When $\tau = 0$ and $\tau = T_k$, the two last terms are 0
- Functional satisfies the boundary conditions $\rightarrow S$ and R do not need to be positive definite.

Stability analysis

Proposed functional

- Starting point [Seuret]

$$
\begin{aligned}
V &= x(t)^T P x(t) + V_1(t) \\
V_1 &= (T_k-\tau)\zeta(t)^T[S\zeta(t)+2Qx(t_k)] + (T_k-\tau)\int_{t_k}^{t}\dot{x}(s)^T R\dot{x}(s)ds \\
&\quad +(T_k-\tau)\tau x(t_k)^T U x(t_k) \\
\zeta(t) &= x(t)-x(t_k)
\end{aligned}
$$

- Fragmentation (Discretization) N pieces ($N+1$ points)

$$
t_k^i(t) = t_k + \frac{N-i}{N}(t-t_k)
$$

$$
\begin{aligned}
\int_{t_k}^{t}\dot{x}(s)^T R\dot{x}(s)ds &\rightarrow \sum_{i=0}^{N-1}\int_{t_k^{i-1}(t)}^{t_k^i(t)}\dot{x}(s)^T R_i\dot{x}(s)ds \\
\zeta(t) &\rightarrow \zeta_k(t) = \underset{i=0,\ldots,N-1}{\mathrm{col}}\{x(t_k^i(t)) - x(t_k^{i-1}(t))\}
\end{aligned}
$$

Stability condition

Theorem

The sampled-data system is asymptotically stable for any time-varying sampling period in $[T_{MIN}, T_{MAX}]$ *if there exist constant matrices* $P = P^T \succ 0$, $R_i = R_i^T \succ 0$, $i = 0, \ldots, N-1$ *and* $U = U^T \in \mathbb{R}^{n \times n}$, $S = S^T \in \mathbb{R}^{nN \times nN}$, $Q \in \mathbb{R}^{nN \times n}$ *and* $Y \in \mathbb{R}^{n(N+1) \times nN}$ *such that the LMIs*

$$\begin{array}{rclrcl} \Psi_1 + T_{MIN}(\Psi_2 + \Psi_3) & \prec & 0, & \Psi_1 + T_{MAX}(\Psi_2 + \Psi_3) & \prec & 0 \\ \begin{bmatrix} \Psi_1 - T_{MIN}\Psi_3 & T_{MIN}Y \\ \star & -\alpha^- \bar{R} \end{bmatrix} & \prec & 0, & \begin{bmatrix} \Psi_1 - T_{MAX}\Psi_3 & T_{MAX}Y \\ \star & -\alpha^+ \bar{R} \end{bmatrix} & \prec & 0 \end{array} \tag{6}$$

hold where $\alpha^- = NT_{MIN}$, $\alpha^+ = NT_{MAX}$.

- Affine in T $\rightarrow$ semi-infiniteness easy to handle
- Affine in the system matrices $\rightarrow$ easy to extend to uncertain system
- No state-transition matrix involved $\rightarrow$ nonlinear systems

Example 1

- Let us consider the system

$$\dot{x}(t) = Ax(t) + BKx(t_k)$$

with

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}, \qquad BK = \begin{bmatrix} 0 & 0 \\ -0.375 & -1.15 \end{bmatrix}$$

- Constant sampling-period $\mathcal{T} = [0, 1.7294]$.

Theorems	Ex.1
[Fridman et al., 04]	$[0,\ 0.869]$
[Naghshtabrizi et al., 08]	$[0,\ 1.113]$
[Fridman et al.,10]	$[0,\ 1.695]$
[Liu et al., 09]	$[0,\ 1.695]$
Proposed result, $N = 1$	$[0,\ 1.721]$
Proposed result, $N = 3$	$[0,\ 1.727]$
Proposed result, $N = 5$	$[0,\ 1.728]$

Example 2

- Let us consider the system

$$\dot{x}(t) = Ax(t) + BKx(t_k)$$

with

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \qquad BK = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}$$

- Constant sampling-period $\mathcal{T} = [0, 3.2716]$

Theorems	Ex.2
[Fridman et al., 04]	$[0,\ 0.99]$
[Naghshtabrizi et al., 08]	$[0,\ 1.99]$
[Fridman et al.,10]	$[0,\ 2.03]$
[Liu et al., 09]	$[0,\ 2.53]$
Proposed result, $N = 1$	$[0,\ 2.51]$
Proposed result, $N = 3$	$[0,\ 2.62]$
Proposed result, $N = 5$	$[0,\ 2.64]$

Example 3

- Let us consider the system

$$\dot{x}(t) = Ax(t) + BKx(t_k)$$

with

$$A = \begin{bmatrix} 0 & 1 \\ -2 & 0.1 \end{bmatrix}, \qquad BK = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

- Constant sampling-period $\mathcal{T} = [0.2007, 2.0207]$

Theorems	Ex.3
[Fridman et al., 04]	-
[Naghshtabrizi et al., 08]	-
[Fridman et al.,10]	-
[Liu et al., 09]	-
Proposed result, $N = 1$	$[0.40,\ 1.11]$
Proposed result, $N = 3$	$[0.40,\ 1.28]$
Proposed result, $N = 5$	$[0.40,\ 1.31]$

Conclusion

Conclusion

- Functional-based approach suitable for stability analysis
- Fragmentation improves results
- Possible extension to uncertain and nonlinear systems

Thank you for your attention !