H_∞ observer design for uncertain time-delay systems

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***Abstract*— This paper is concerned with robust observer design for linear time-delay systems in the presence of unstructured uncertainties and of disturbances as well. The proposed method ensures the stability of the observer and the H_∞ attenuation of uncertainty and disturbance effects on the estimated error. The result is developed ina delay dependent framework and the observer gain matrix is obtained by solving a linear matrix inequality (LMI).**

***Index Terms*— Observers, Time-delay, H_∞ approach, uncertainties**

I. INTRODUCTION

Analysis and control of systems with delays have been subject to lots of works and studies (see for example [1], [2], [3]). Among those contributions, several observer schemes for linear systems with time-delays have been proposed in the literature [4], [5], [6], [7]. In the H_∞ framework, observers for time-delay systems have been developed for instance in [8], [9], [10] with a disturbance reduced effects on the estimated error, by solving some modified algebraic Riccati equations. However, the previous results do not include any robustness constraints, while modelling errors may destabilize the observer practically if they are not taken into account when designing the observer [11]. Other results do exist in robust filtering for time-delay systems [12], [13], but do not consider the presence of control input, and therefore cannot be applied to the case of input uncertainties.

Very few contributions concern robust observer design. In [14], [15] the Lyapunov-Krasovskii theory (in the delay independent framework) is used to deal with norm-bounded uncertainties on each of the system matrices. In both papers, the uncertainties on each system matrix M are assumed to be of the form $\Delta M(.) = HF(.)N$ where $(.)$ means that this could be time-varying uncertainties, H and N are known constant matrices, and F, which may be time-varying, is unknown and meets $F^T F \leq I$. In [14] the design of robust observer-based H_∞ control is obtained through the resolution of two coupled Riccati equations, which extends the results of [10] to uncertain systems (at the price of complexity). In the same way, in [15], the solution is obtained by solving two Riccati equations. In fact, if both contributions are interesting, numerical problems may arise in solving these complicated Riccati equations. Moreover the case of delay uncertainties is not tackled in those works which makes their application quite unrealistic.

To our best knowledge, only the previous works of the authors has been concerned with the design of robust observers in the case of uncertainties including delay uncertainties. First, in [16] the parametrization of all stable observers for a time-delay system is used to obtain a parameterization matrix that solves an optimization problem such that the effects of *unstructured uncertainties* on the estimated states is minimized. A suboptimal solution for this optimization problem has been proposed only for spectrally co-canonical time-delay systems. However the proposed procedure needs excessive calculations, in particular to obtain the parametrization of all stable observers, which makes this method difficult to use. Then in [17] the case of additive and multiplicative input unstructured uncertainties is considered and a robust observer is then developed in the delay independent framework using an LMI approach.

This paper is concerned with uncertain linear systems with time-delay, submitted to some external disturbance input. The main contribution of this paper is then to provide a robust H_∞ observer design LMI solution in the case of frequency-domain unstructured uncertainties (allowing to handle delay uncertainties). It follows [17] and provides some important extensions:

- The class of considered systems is much more important as they are submittted to uncertainties and external unknown disturbance as well.
- The proposed methodology allows to tackle all uncertainty types while in [17] only additive and multiplicative input uncertainties were considered
- The observation error stability and the attenuation of uncertainties and disturbance effects are developed within a delay dependent H_∞ framework, allowing to reduce the conservatism of the approach in [17].
- The minimisation of both uncertainty and disturbance effects is shown within the concept of Pareto optimality.

The paper is organized as follows. The system and uncertainty modelling are described in Section 2. The problem statement is presented in section 3. Section 4 is devoted to design a robust observer for time-delay systems. Two illustrative examples are given in Section 5. The paper concludes with Section 6.

Notations:

$\mathbb{R}$ is the field of real numbers, $\mathbb{N}$ is the set of integer numbers, s denotes the Laplace variable, $z = e^{-sh}$ and $h \in \mathbb{R}^+$ fixed, $\forall\, x \in \mathbb{R}^n$, $x_h = x(t-h)$.

$\|.\|_2$ is the H_2-norm defined by: $\|X(s)\|_2 = [\frac{1}{2\pi}\int_{-\infty}^{\infty} X'(-j\omega)X(j\omega)d\omega]^{\frac{1}{2}}$, $\|.\|_\infty$ is the H_∞-norm defined by the induced H_2 vector norm.

$\mathscr{C}[a,b]$ is the set of continuous functions $[a,b] \to \mathbb{R}^n$, I_n denotes the $(n \times n)$ identity matrix.

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II. Model and uncertainty description

Consider the time-delay system G described by the nominal model:

$$\left\{\begin{array}{lcl}\dot{x}(t) & = & A_0x(t)+A_1x(t-h)+Bu_o(t)+Ed(t)\\ y_o(t) & = & C_0x(t)+C_1x(t-h)\\ x(t) & = & \psi(t); \qquad t\in[-h,0]\end{array}\right. \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $u_o(t)\in\mathbb{R}^r$ is the nominal control input vector and $d(t)\in\mathbb{R}^q$ the unknown L_2-bounded disturbance input, $y_o(t)\in\mathbb{R}^p$ is the nominal output vector, A_i, B_i, C_i, $i=0,1,\cdots,m$ ($m\in\mathbb{N}^+$) are constant matrices with appropriate dimensions, $\psi(t)\in\mathscr{C}[-mh,0]$ is the functional initial condition of (1), $h\in\mathbb{R}^+$ is the time-delay.
The input/output transfer matrix of system (1) is given by:

$$G(s,z)=C(z)(sI_n-A(z))^{-1}B(z) \tag{2}$$

where $z=e^{-sh}$, $A(z)=A_0+A_1z$ and $C(z)=C_0+C_1z$.
The uncertainties are here considered on the input/output behavior (i.e. no uncertainties are included in the disturbance/output transfer matrix, as it could be considered as a new disturbance input).

Definition 1: Let $G(s,z)$ be the nominal model (1) of the real plant $\tilde{G}(s,z)=f(G(s,z),\Delta(s,z))$, and $\Delta(s,z)$ a variable stable transfer matrix s.t. $\|\Delta(s,z)\|_\infty\leq\delta$. According to figures (1- 5) $\Delta(s,z)$ represents unstructured uncertainties which may have different forms [18]:

$$\begin{array}{lcl}Fig.1 & : & \tilde{G}=G+\Delta_a, \|\Delta_a\|_\infty\leq\delta_a\\ Fig.2 & : & \tilde{G}=G(I_r+\Delta_I), \|\Delta_I\|_\infty\leq\delta_I\\ Fig.3 & : & \tilde{G}=G(I_r+\Delta_{iI})^{-1}, \|\Delta_{iI}\|_\infty\leq\delta_{iI}\\ Fig.4 & : & \tilde{G}=(I_p+\Delta_O)G, \|\Delta_O\|_\infty\leq\delta_O\\ Fig.5 & : & \tilde{G}=(I_p+\Delta_{iO})^{-1}G, \|\Delta_{iO}\|_\infty\leq\delta_{iO}\end{array} \tag{3}$$

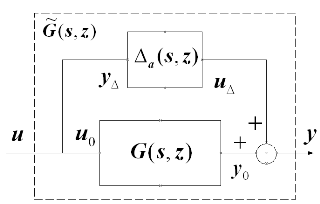

Fig. 1. Additive uncertainty.

Following these uncertainty types, the system description should be modified in such a way:

$$\tilde{G}\left\{\begin{array}{lcl}\dot{x}(t) & = & A_0x(t)+A_1x(t-h)+Bu(t)\\ y(t) & = & y_o(t)+u_\Delta(t),\end{array}\right. \tag{4}$$

for additive and direct multiplicative output uncertainties, and in a similar way for:

- Inverse multiplicative output uncertainties : $u(t)=u_o(t)$, $y(t)=y_o(t)-u_\Delta(t)$
- Direct multiplicative input uncertainties : $u(t)=u_o(t)-u_\Delta$, $y(t)=y_o(t)$
- Inverse multiplicative input uncertainties : $u(t)=u_o(t)+u_\Delta$, $y(t)=y_o(t)$

Assumption 1: The uncertainties are assumed to be L_2-bounded, i.e. $\|u-u_o\|_2$ is assumed to be bounded (or $\|y-y_o\|_2$ according to the uncertainty types).

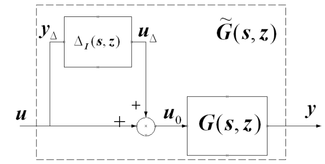

Fig. 2. Mult. input unc.

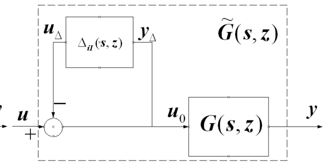

Fig. 3. Inv. mult. input unc.

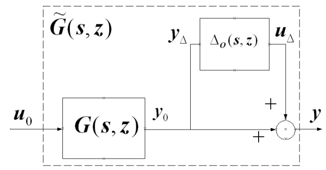

Fig. 4. Mult. output unc.

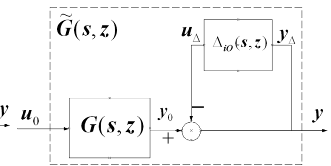

Fig. 5. Inv. mult. output unc.

Remark 1: Note that, for additive and input uncertainties, this hypothesis only implies that finite escape-time of the uncertainties will not occur, but does not avoid signals to diverge. For output uncertainties it remains to require a prior plant feedback stabilization which, in practice, is necessary for the implementation of observer.

Remark 2: The proposed uncertainty description allows to include delay uncertainties, which is new compared to the existing approaches, where uncertainties are generally considered on the system matrices (s.t ΔA ...) but not on the system time-delay. Indeed if the real delay is different from the nominal one, then the corresponding uncertainty can be included for instance in the representation (4). This will be illustrated in section 5.

III. Problem statement

Let us now consider the following full order Luenberger-type observer for the nominal system (1):

$$\left\{\begin{array}{lcl}\dot{\hat{x}} & = & A_0\hat{x}+A_1\hat{x}_h+Bu_o+L\ (y_o-\hat{y})\\ \hat{y} & = & C_O\hat{x}+C_1\hat{x}_h\\ \hat{r} & = & D\hat{x}\end{array}\right. \tag{5}$$

where $\hat{x}(t)\in\mathbb{R}^n$, $r(t)\in\mathbb{R}^k$, D is a real $(k\times n)$ matrix, and L is the observer gain to be designed.

Remark 3: Note that this kind of estimation can be useful for instance when an observer is used in a state feedback $K\hat{x}(t)$. Indeed, only $Kx(t)$ should meet robust estimation requirements (which is less conservative than requiring H_∞ performance for the whole state estimation).

When applying the observer on the real plant $\tilde{G}$, (u_o,y_o) has to be replaced by (u,y) in the observer equation (5), i.e. the observer equations become:

$$\left\{\begin{array}{lcl}\dot{\hat{x}} & = & A_0\hat{x}+A_1\hat{x}_h+Bu+L\ (y-\hat{y})\\ \hat{y} & = & C_O\hat{x}+C_1\hat{x}_h\\ \hat{r} & = & D\hat{x}\end{array}\right. \tag{6}$$

The following definition for robust observation can be provided.

Definition 2: Given a time-delay plant $\tilde{G}(s,z)$ of the form (3) with unstructured uncertainties, and its corresponding nominal model (1). Consider the Luenberger-type observer (6). Then $\hat{r}(t)$ is said to be a robust H_∞ estimation of $r(t) = Dx(t)$ if:

1. $\lim_{t\to+\infty}(r(t)-\hat{r}(t)) = 0$ when $\Delta(s) \equiv 0$,
2. Under zero initial conditions, there exists some positive scalars γ_1 and γ_2 s.t., under $\|\Delta(s)\| \leq \delta$:

$$\frac{\|\hat{r}(s)-r(s)\|_2}{\|u_\Delta(s)\|_2} \leq \frac{\gamma_1}{\delta} \quad , \quad \frac{\|\hat{r}(s)-r(s)\|_2}{\|d(s)\|_2} \leq \gamma_2$$

Notice that first condition of Definition 1 ensures the nominal stability of the observer while the second condition ensures its robustness and performance.

Both conditions of Definition 1 are analyzed below.

Analysis of Condition 1: Consider system (1) and observer (5) (i.e. assume that $\Delta(s) \equiv 0$). The nominal estimation error is then such that:

$$\dot{e} = (A_0 - LC_0)e + (A_1 - LC_1)e_h \tag{7}$$

If L is designed to ensure the stability of the previous time-delay system, then we have:

$$\lim_{t\to\infty}(r(t)-\hat{r}(t)) = \lim_{t\to\infty} De(t) = 0$$

and Condition 1 in Definition 2 is satisfied.

Remark 4: Notice that, in the next section, stability of the observer is proved in a delay dependent framework. This implies that observability of the pair $(A_0 + A_1, C_0)$ is a necessary condition for the observer design.

Analysis of Condition 2: Consider system (1) and observer (6) and assume that $\Delta(s) \not\equiv 0$. The real estimation error $e(t) := x(t) - \hat{x}(t)$, satisfies different dynamical equations according to the uncertainty type. For additive, direct or inverse multiplicative output uncertainties:

$$\dot{e} = (A_0 - LC_0)e + (A_1 - LC_1)e_h + Ed \pm Lu_\Delta, \tag{8}$$

where $\pm$ is either $+$ or $-$ according to the uncertainty type. For direct or inverse multiplicative input uncertainties.

$$\dot{e} = (A_0 - LC_0)e + (A_1 - LC_1)e_h + Ed \pm Bu_\Delta, \tag{9}$$

In the frequency domain, the real estimation error $e_r(t) = r(t) - \hat{r}(t) = De(t)$ is then given by:

$$e_r(s) = \pm D(sI_n - A(z) + LC(z))^{-1}Lu_\Delta(s), \tag{10}$$

or

$$e_r(s) = \pm D(sI_n - A(z) + LC(z))^{-1}Bu_\Delta(s). \tag{11}$$

Defining

$$\begin{cases} T_a(s,z) &= D(sI_n - A(z) + LC(z))^{-1}L \\ T_I(s,z) &= D(sI_n - A(z) + LC(z))^{-1}B \\ T_d(s,z) &= D(sI_n - A(z) + LC(z))^{-1}E \end{cases} \tag{12}$$

we have

$$\|e_r(s)\|_2 = \|T_a(s,z)\|_\infty \|u_\Delta(s)\|_2 + \|T_d(s,z)\|_\infty d(s) \tag{13}$$

or

$$\|e_r(s)\|_2 = \|T_I(s,z)\|_\infty \|u_\Delta(s)\|_2 + \|T_d(s,z)\|_\infty d(s) \tag{14}$$

and From (13-14), we can conclude that if the uncertainty output vector u_Δ is bounded, then $e_r(t)$ will remain bounded as transfer matrices $T_I(s,z)$, $T_a(s,z)$ and $T_d(s,z)$ are stable. In addition, if the H_∞ norm of the transfer matrices $T_a(s,z)$ (or $T_I(s,z)$) and $T_d(s,z)$ are minimized, then $\|e_r(s)\|_2/\|u_\Delta\|_2$ and $\|e_r(s)\|_2/\|d\|_2$ will be minimized and Condition 2 in Definition 2 is satisfied.

Remark 5: Note that, as $\Delta(s,z)$ is assumed to be stable and the input $u_\Delta(t)$ is assumed to be bounded, the dynamic of the observer remains stable when it is applied on the real system. The estimation error will not be asymptotically stable but will be bounded.

IV. Robust observer design

This section is devoted to the design of a robust estimation of $r(t)$ via a Linear Matrix Inequality approach according to Definition 1. The aim of this part is to give a Bounded Real Lemma in the case of unstructured uncertainties and disturbance input as well. The result is developed in a delay-dependent framework. In all this section, Assumption 1 is assumed to be fulfilled and the uncertainties are assumed to be weighted, i.e. we will note:

$$w_\Delta = W(s)\Delta(s)y_\Delta \tag{15}$$

$$u_\Delta = \Delta(s)y_\Delta \tag{16}$$

with $\|W(s)\| \leq \delta$ and $\|\Delta(s)\| \leq 1$.

Many results have been devoted in the literature to the stability and stabilization of time-delay systems within a delay dependent framework. Let us cite for instance [19], [20]. We have considered in this paper the methodology developed in [20] (which concerns systems with time-varying delays) as it is quite simple to implement and few conservative. However, as the results only concern stability, we have extended it in an H_∞ framework.

The first main result is the following:

Proposition 1: Let us consider a time-delay system $\dot{z}(t) = A_0 z(t) + A_1 z(t-\tau(t)) + Ed + B_\Delta w_\Delta$, $q(t) = Dz(t)$, s.t. $0 \leq \tau(t) \leq \tau_{max}$ and where d and w_Δ are L_2 signals. Given positive scalars δ, γ_1 and γ_2, the above system is L_2 stable if there exist symmetric positive definite matrices $P = P^T > 0$, $Q = Q^T > 0$, a symmetric semi-positive-definite $X = \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix} \geq 0$, some matrices of appropriate dimensions Y, T and Z such that the following LMIs are true:

$$\Phi < 0 \text{ and } \Psi \geq 0 \tag{17}$$

with

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \tau_{max}A_0^T Z & PB_\Delta & PE \\ * & \Phi_{22} & \tau_{max}A_h^T Z^T & 0 & 0 \\ * & * & -\tau_{max}Z & 0 & 0 \\ * & * & * & -(\frac{\gamma_1}{\delta})^2 I_r & 0 \\ * & * & * & * & -\gamma_2^2 I_q \end{bmatrix}, \tag{18}$$

$$\Psi = \begin{bmatrix} X_{11} & X_{12} & Y \\ * & X_{22} & T \\ * & * & Z \end{bmatrix}, \tag{19}$$

and

$$\Phi_{11} = PA_0 + A_0^T P + Y + Y^T + Q + \tau_{max} X_{11} + D^T D \quad (20)$$
$$\Phi_{12} = PA_h - Y + T^T + \tau_{max} X_{12} \quad (21)$$
$$\Phi_{22} = -T - T^T - (1-\nu)Q + \tau_{max} X_{22} \quad (22)$$

where $*$ means the symmetric element.

Sketch of proof: The proof is a direct extension of the one in [20] to the case of systems with unknown input. Using the proof of Theorem 2 in [20], the following Lyapunov functional is considered:

$$V(z_t) = z^T(t)Pz(t) + \int_{t-\tau(t)}^{t} z^T(s)Sz(s)ds \quad (23)$$
$$+ \int_{-\tau_{max})}^{0} \int_{t+\theta}^{t} \dot{z}^T(s)Z\dot{z}(s)dsd\theta \quad (24)$$

Let us introduce the H_∞ criterion:

$$J := \int_0^\infty [q^T(t)q(t) - (\frac{\gamma_1}{\delta})^2 w_\Delta^T(t)w_\Delta(t) - \gamma_2^2 d^T(t)d(t)]dt$$

then,

$$J \leq \int_0^\infty [q^T q - (\frac{\gamma_1}{\delta})^2 w_\Delta^T w_\Delta - \gamma_2^2 d^T d + \dot{V}]dt$$

Using the same developments of the proof in [20] leads to LMIs (17).

Note that using (15), $J \leq 0$ leads to:

$$\|\frac{q(s)}{d(s)}\| \leq \gamma_2$$

and

$$\|\frac{q(s)}{u_\Delta(s)}\| \leq \frac{\gamma_1}{\delta}\|\Delta(s)\|_\infty \leq \gamma_1$$

□

The above proposition is then applied on the observation error system to get the following main result.

Theorem 1: Given positive scalars δ, α, γ_1 and γ_2, (6) is a robust H_∞ observer for system (4) according to Definition 2, if there exist symmetric positive definite matrices $P = P^T > 0$, $Q = Q^T > 0$, a symmetric semi-positive-definite $X = \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix} \geq 0$, some matrices of appropriate dimensions Y, T and Z such that the following LMIs are true:

$$\Phi < 0 \text{ and } \Psi \geq 0 \quad (25)$$

with

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \tau_{max}\alpha A_0^T P & PB_\Delta & PE \\ * & \Phi_{22} & \tau_{max}\alpha A_h^T P^T & 0 & 0 \\ * & * & -\tau_{max}\alpha P & 0 & 0 \\ * & * & * & -(\frac{\gamma_1}{\delta})^2 I_r & 0 \\ * & * & * & * & -\gamma_2^2 I_q \end{bmatrix}, \quad (26)$$

$$\Psi = \begin{bmatrix} X_{11} & X_{12} & Y \\ * & X_{22} & T \\ * & * & \alpha P \end{bmatrix}, \quad (27)$$

and

$$\Phi_{11} = PA_0 + A_0^T P + KC + K^T C^T + \quad (28)$$
$$Y + Y^T + Q + \tau_{max} X_{11} + D^T D \quad (29)$$
$$\Phi_{12} = PA_h + KC_h - Y + T^T + \tau_{max} X_{12} \quad (30)$$
$$\Phi_{22} = -T - T^T - (1-\nu)Q + \tau_{max} X_{22} \quad (31)$$

The observer gain is then given by:

$$L = -P^{-1}K \quad (32)$$

Sketch of proof: The development consist in applying Proposition 1 on the estimation error system (8) or (9). For simplicity B_Δ denotes either L or E according to the uncertainty type (see (8)- (9)).

Then the obtained LMI is non linear as it includes the variables PL and ZL. In order to linearize the inequality, we have used the idea in [21], i.e. to choose:

$$Z = \alpha\ P, \quad \alpha \in \mathbb{R}$$

Then L satisfies:

$$L = -P^{-1}K$$

Note that, in the case when $B_\Delta = L$ then the term PB_Δ in the LMI (18) must be replaced by $-K$. □

If the minimal attenuation bound is required (for the uncertainties subject to fixed disturbance attenuation level) the following optimization problem have to be be solved:

$$\gamma_{min}^2 = \min_{P,Q,X,Y,T,Z} \gamma_1^2 \quad (33)$$
$$\text{s.t. } \Phi < 0, \Psi \geq 0, P > 0, Q > 0, X > 0, \gamma_2 > 0$$

Of course the reverse optimization problem (i.e. minimize γ_2 subject to fixed γ_1) could be also tackled, as presented in the exemple later. Both minimisation problems can allow to exhibit some Pareto optimal couple (γ_1, γ_2), as we will see in the example section.

V. Illustrative example

Consider the unstable time-delay system:

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t-h) \\ \qquad + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} d(t) \\ y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) \end{cases}$$

The system matrices A_0, A_1 and B are assumed to be uncertain matrices with 10% of uncertainties. The constant delay is assumed to be in the interval $h \in [0;3]$ with a nominal value $h = 1.5sec$.

To illustrate the proposed methodology, the case additive uncertainties is considered.

In this case, the transfer matrix can be represented as: $\tilde{G}(s,z) = \{G(s,z) + W(s)\Delta(s,z) : \|\Delta\|_\infty \leq 1\}$, with

$$G(s,z) = \frac{1}{s^2 + s - z}, \qquad W(s) = \frac{s + 1.35}{1.5s^2 + 0.8s + 2.5}$$

Note that $\Delta_a = W\Delta$ is such that $\|\Delta_a\|_\infty \leq \delta_a := 2.6$.

The additive uncertainties and the chosen weighting function

$W(s)$ are represented in figure 6. As seen in figure 7, when the delay is assumed to be constant and the other parameters only can change, the uncertainties are lower. Then the maximum of uncertainties in figure 6 is mainly due to the uncertain delay which emphasizes the interest for frequency domain uncertainty description.

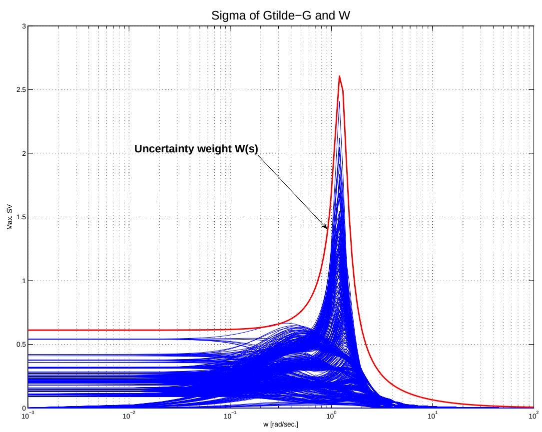

Fig. 6. Bode diagram of $\tilde{G}(s,z)$ -$G(s,z)$.

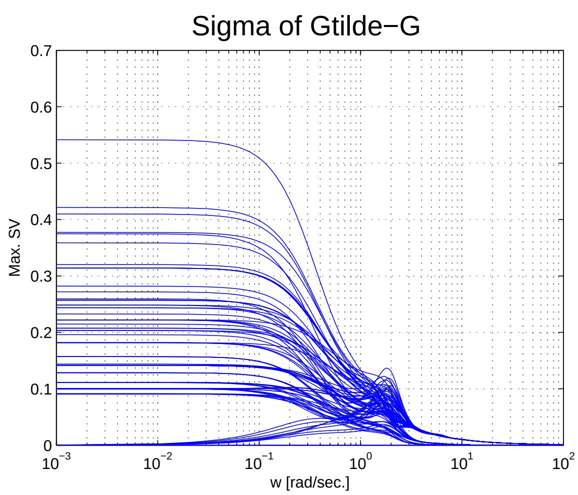

Fig. 7. Parameter uncertainties (constant delay $h = 1.5$.).

In order to design a robust observer for $x(t)$ the proposed method is applied and Theorem 1 is solved.

First the attenuation γ_1 is fixed and we look for the minimal disturbance attenuation level γ_2 and then reciprocally. To emphasize the Pareto optimality, the optimisation problem is solved for different γ_1 as illustrated on figure 8 where the obtained minimal γ_2 is plot versus γ_1, and vice versa. Note that both cases lead to the same result (i.e. the two curves in Fig 8 are equals), which shows the efficiency of the optimization by LMIs.

The Pareto limit in figure 8 emphasizes the usual performance/robustness trade-off, as the minimisation of γ_1 will lead to an increase of γ_2.

For simulation purpose, we have chosen the couple $(\gamma_1,\ \gamma_2) =$

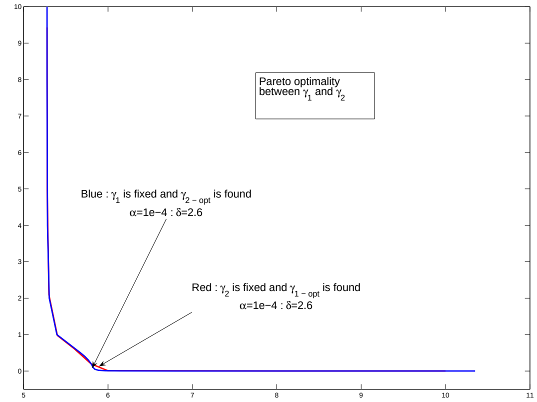

Fig. 8. Pareto optimality, xaxis=γ_1, yaxis = γ_2

$(5.8435,\ \ 0.05)$, as it is not so far from the minimal uncertainty attenuation (~ 5.277) and provides good disturbance attenuation. The observer gain is then $L = \begin{bmatrix} 101.74 & 99.12 \end{bmatrix}^T$. The simulation results are shown in figure 9 where the uncertainties are added in the additive form.

For the simulation an initial value at time $t = -1.5sec$ is used to generate an initial value function on $t \in [-1.5, 0]$. We can see that the effects of the uncertainties and disturbance have been quite attenuated which proves the performance and robustness of the observer.

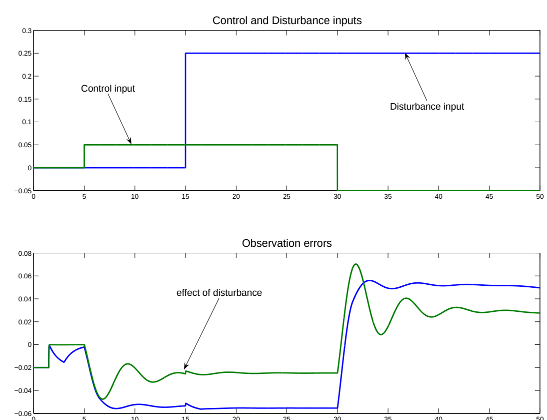

Fig. 9. Simulation.

Figure 10 shows the singular values of $D(j\omega I_n - A(e^{-j\omega}) + LC(e^{-j\omega}))^{-1}LW(j\omega)$ versus the frequency and figure 11 shows the singular values of $D(j\omega I_n - A(e^{-j\omega}) + LC(e^{-j\omega}))^{-1}E$ versus the frequency. These plots represent respectively the effects of the system uncertainty and of the disturbance on the real estimation. It shows that $\|T_a(s,z)\|_\infty < \gamma_1 = 5.8$ and $\|T_d(s,z)\|_\infty < \gamma_2 = 0.05$.

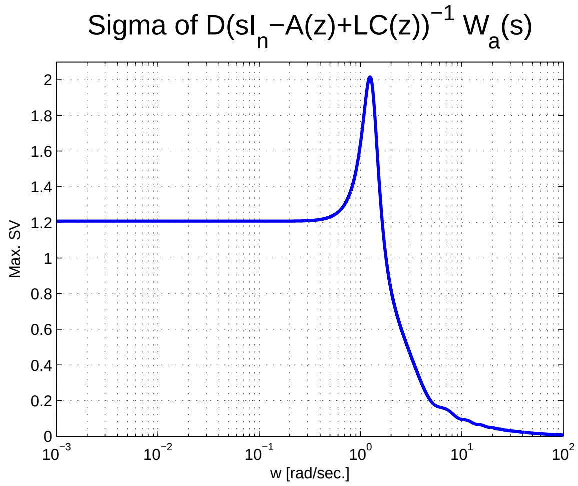

Fig. 10. Effects of the uncertainty on the estimated error.

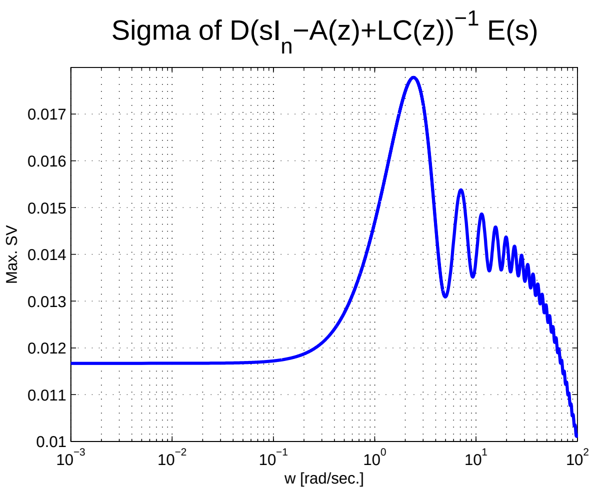

Fig. 11. Effect of disturbance on the estimation error

VI. Conclusion

In this paper, an LMI approach has been developed to design robust H_∞ observers for linear time-delay systems, in a delay dependent framework. The method ensures the stability of the observer and the attenuation of unstructured uncertainties effects and of disturbance effect as well on the estimated error. It is worth noting that this result provides a new formulation, easily reusable, to the design of robust observers, which is quite important in practical situations. The concept of Pareto optimality has been used to emphasize, in the optimisation, the performance/robustness trade-off.
The extension to multiple time-delay systems systems is straightforward. However when the state, input and output delays are different the issue is not easy, which may be a short-term study. Further work may also concern the mixed design of robust H_∞ observer-controllers for time-delay systems w.r.t uncertainties.

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